



# Ideal Efficiencies

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## 1. INTRODUCTION

In this chapter we deal with the simplest ideas that have been used in the past to attain an understanding of solar cell efficiencies from a theoretical point of view. The first and most obvious attack on this problem is to use thermodynamics, and we offer four such estimates in [Section 2](#). Only the first of these is the famous Carnot efficiency. The other three demonstrate that one has more possibilities even within the framework of thermodynamics. To make progress, however, one has to introduce at least one solid-state characteristic, and the obvious one is the energy gap,  $E_g$ . That this represents an advance in the direction of a more realistic model is obvious, but it is also indicated by the fact that the efficiency now calculated is lower than the (unrealistically high) thermodynamic efficiencies ([Section 3](#)). In order to get closer to reality, we introduce in [Section 4](#) the fact that the radiation is effectively reduced from the normal blackbody value (Equation (6)) owing to the finite size of the solar disc. This still leaves important special design features such as the number of series-connected tandem cells and higher-order impact ionisation, and these are noted in [Section 5](#).



## 2. THERMODYNAMIC EFFICIENCIES

The formulae for ideal efficiencies of solar cells are simplest when based on purely thermodynamic arguments. We here offer four of these: they involve only (absolute) temperatures:

- $T_a$ , temperature of the surroundings (or the ambient),
- $T_s$ , temperature of the pump (i.e., the sun),
- $T_c$ , temperature of the actual cell that converts the incoming radiation into electricity.

From these temperatures, we form the following efficiencies [1]:

$$\eta_C \equiv 1 - T_a/T_s, \text{ the Carnot efficiency} \quad (1)$$

$$\eta_{CA} \equiv 1 - (T_a/T_s)^{\frac{1}{2}}, \text{ the Curzon–Ahlborn efficiency} \quad (2)$$

$$\eta_L \equiv 1 - (4/3)(T_a/T_s) + (1/3)(T_a/T_s)^4, \text{ the Landsberg efficiency} \quad (3)$$

$$\eta_{PT} = [1 - (T_c/T_s)^4][1 - T_a/T_c], \text{ the photo–thermal efficiency} \quad (4)$$

due to Müser

In the latter efficiency, the cell temperature is determined by the quintic equation

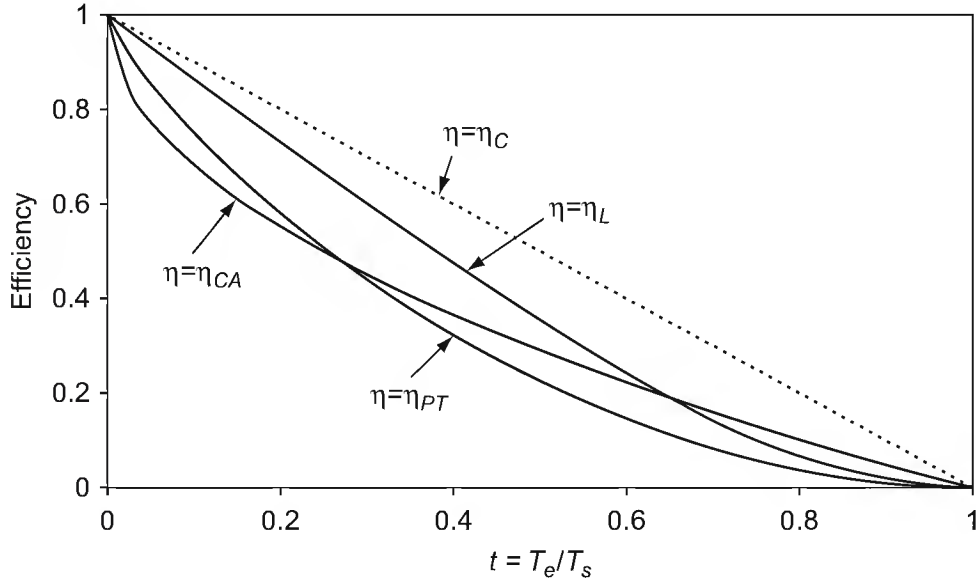
$$4T_C^5 - 3T_aT_c^4 - T_aT_s^4 = 0 \quad (5)$$

The names associated with these efficiencies are not historically strictly correct: for example, in Equations (2) and (3) other authors have played a significant part.

Figure 1 [1] shows curves of the four efficiencies, which all start at unity when  $T_a/T_s \equiv 0$ , and they all end at zero when  $T_a = T_s$ . No efficiency ever beats the Carnot efficiency, of course, in accordance with the rules of thermodynamics. Values near  $T_s = 5760\text{--}5770\text{ K}$  seem to give the best agreement with the observed solar spectrum and the total energy received on Earth, but a less accurate but more convenient value of  $T_s = 6000\text{ K}$  is also commonly used. Using the latter value of  $T_s$  and  $T_a = 300\text{ K}$  as the temperature for Earth, one finds

$$\eta_C = 95\%, \eta_{CA} = 77.6\%, \eta_L = 93.3\%, \eta_{PT} = 85\%$$

If  $T_s = T_a = T_c$  one has in effect an equilibrium situation, so that the theoretical efficiencies are expected to vanish.



**Figure 1** The efficiencies (1)–(4) as functions of  $T_a/T_s$ .

The above thermodynamic efficiencies utilise merely temperatures, and they lie well above experimental results. One needs an energy gap ( $E_g$ ) as well to take us from pure thermodynamics to solid-state physics. Incident photons can excite electrons across this gap, thus enabling the solar cell to produce an electric current as the electrons drop back again. The thermodynamic results presented earlier, on the other hand, are obtained simply by considering energy and entropy fluxes.



### 3. EFFICIENCIES IN TERMS OF ENERGIES

In order to proceed, we need next an expression for the number of photons in blackbody radiation with an energy in excess of the energy gap,  $E_g$  say, so that they can excite electrons across the gap. At blackbody temperature  $T_s$  the number of photons incident on unit area in unit time is given by standard theory as an integral over the photon energy [2]:

$$\Phi(E_g, T_s) = \frac{2\pi k^3}{h^3 c^2} T_s^3 \int_{E_g/kT_s}^{\infty} \frac{x^2 dx}{e^x - 1} \quad (6)$$

Now suppose that each of these photons contributes only an energy equal to the energy gap to the output of the device, i.e., a quantity proportional to

$$x_g \int_{x_g}^{\infty} \frac{x^2 dx}{e^x - 1} \quad (x_g \equiv E_g/kTs) \quad (7)$$

To obtain the efficiency  $\eta$  of energy conversion, we must divide this quantity by the whole energy that is, in principle, available from the radiation:

$$\eta = x_g \int_{x_g}^{\infty} \frac{x^2 dx}{e^x - 1} / \int_0^{\infty} \frac{x^2 dx}{e^x - 1} \quad (8)$$

Equation (8) gives the first of the Shockley–Queisser estimates for the limiting efficiency of a solar cell, the *ultimate efficiency* (see Figure 5). The argument neglects recombination in the semiconductor device, even radiative recombination, which is always present (a substance that absorbs radiation can always emit it!). It is also based on the blackbody photon flux (Equation (6)) rather than on a more realistic spectrum incident on Earth.

We shall return to these points in Section 4, but first a brief discussion of Equation (8) is in order. There is a maximum value of  $\eta$  for some energy gap that may be seen by noting that  $\eta = 0$  for both  $x_g = 0$  and for  $x_g$  very large. So there is a maximum efficiency between these values. Differentiating  $\eta$  with respect to  $x_g$  and equating to zero, the condition for a maximum is

$$x_g = x_{gopt} = 2.17$$

corresponding to  $\eta = 44\%$ .

This is still higher than most experimental efficiencies, but the beauty of it is that it is a rather general result that assumes merely properties of blackbody radiation.

Let  $f(x)$  be a generalised photon distribution function; then a generalised efficiency can be defined by

$$\eta = \frac{x_g \int_{x_g}^{\infty} f(x) dx}{\int_0^{\infty} x f(x) dx} \quad (9)$$

The maximum efficiency with respect to  $x_g$  is then given by

$$x_{gopt} f(x_{gopt}) = \int_{x_{gopt}}^{\infty} f(x) dx \quad (10)$$

This is rather general and will serve also when the photon distribution departs from the blackbody forms and even for radiation in different numbers of dimensions.

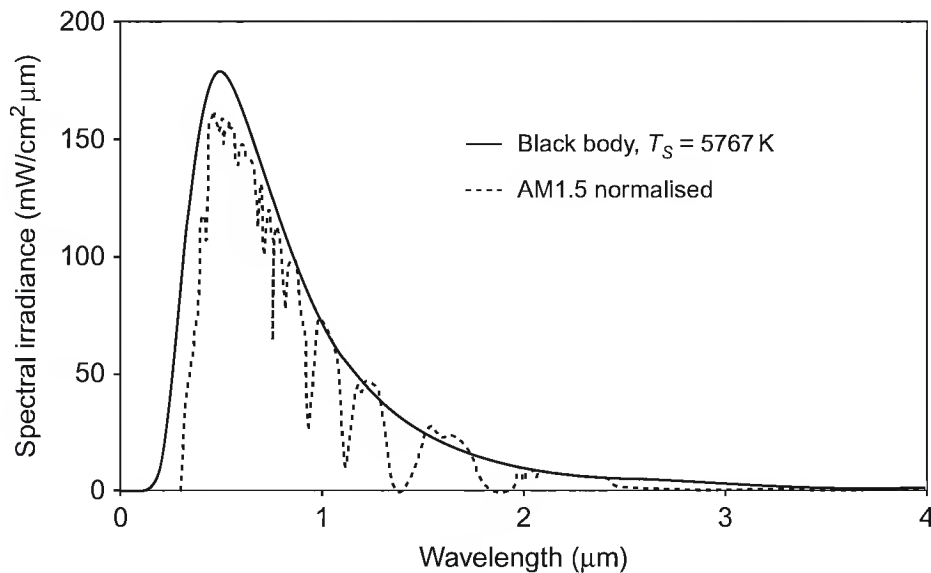


#### 4. EFFICIENCIES USING THE SHOCKLEY SOLAR CELL EQUATION

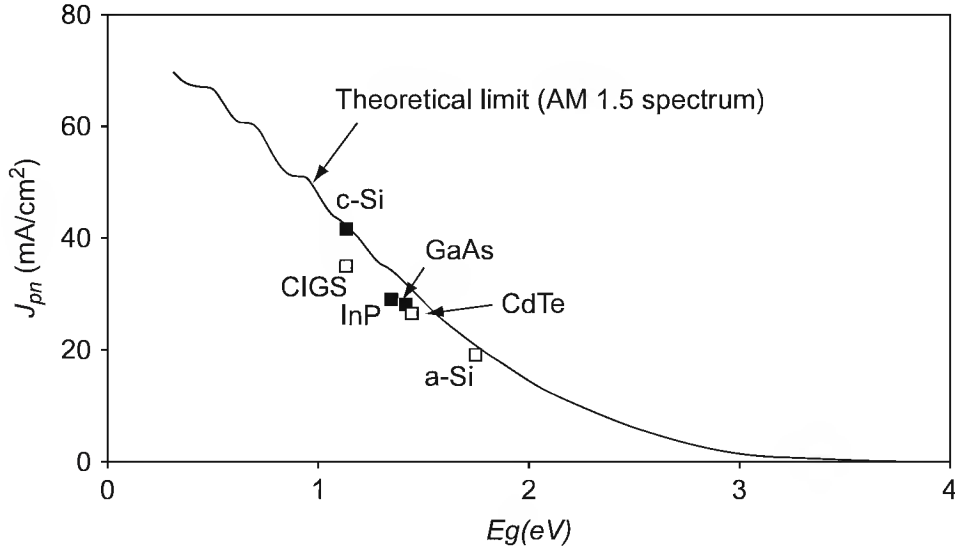
A further step in finding the appropriate efficiency limits for single-junction solar cells can be made by estimating the relevant terms in the Shockley ideal solar cell equation (Equation (1) in Chapter Ia-1). To this end, further remarks must be made about the solar spectrum and solar energy incident on Earth's surface. The ultimate efficiency, discussed in Section 3, was based on the blackbody photon flux (Equation (6)), a rigorous thermodynamic quantity but not a very good estimate of the solar spectrum as seen on Earth. By virtue of the large distance between the Sun and Earth, the radiative energy incident on Earth's surface is less than that of Equation (6) by a factor  $f_\omega$ , which describes the size of the solar disk (of solid angle  $\omega_s$ ) as perceived from Earth:

$$f_\omega = \left( \frac{R_s}{R_{SE}} \right)^2 = \frac{\omega_s}{\pi} \quad (11)$$

where  $R_s$  is the radius of the Sun ( $696 \times 10^3$  km), and  $R_{SE}$  is the mean distance between the Sun and Earth ( $149.6 \times 10^6$  km), giving  $\omega_s = 6.85 \times 10^{-5}$  sterad and  $f_\omega = 2.18 \times 10^{-5}$ . The resulting spectrum is shown in Figure 2 alongside the standard terrestrial AM1.5 spectrum (a further discussion of the



**Figure 2** The blackbody spectrum of solar radiation and the AM1.5 spectrum, normalised to total irradiance  $1 \text{ kW/m}^2$ , which is used for the calibration of terrestrial cells and modules.



**Figure 3** The theoretical limit on photogenerated current, compared with the best measured values. The curve is obtained by replacing the product  $f_{\omega}\Phi(E_g, T_s)$  in Equation (12) by the appropriate AM1.5 photon flux. Full symbols correspond to crystalline materials, open symbols to thin films.

spectra that are used for solar cell measurements in practice can be found in Chapter III-2, which also shows the extraterrestrial spectrum AMO).

The maximum value of the photogenerated current  $I_{ph}$  now follows if we assume that one absorbed photon contributes exactly one electron to the current in the external circuit:

$$I_{ph} = Aqf_{\omega}\Phi(E_g, T_s) \quad (12)$$

where  $A$  is the illuminated area of the solar cell and  $q$  is the electron charge. The maximum photogenerated current density  $J_{ph} = I_{ph}/A$  that can be produced by a solar cell with band gap  $E_g$  is shown in Figure 3. To allow a comparison with photocurrents measured in actual devices, Figure 3 is plotted for the AM1.5 solar spectrum, which is used for calibration of terrestrial solar cells, rather than for the blackbody spectrum used in Section 3.

The open-circuit voltage  $V_{oc}$  can now be obtained using the photogenerated current  $I_{ph}$  (Equation (12)) and the (dark) saturation current  $I_0$  that appears in the ideal solar cell equation:

$$V_{oc} = \frac{kT}{q} \ln \left( 1 + \frac{I_{ph}}{I_0} \right) \quad (13)$$

The current  $I_0$  can be obtained by a similar argument as the photogenerated current  $I_{ph}$ , since, as argued by Shockley and Queisser, it can